Teacher notes

Topic D

Making planets

A planet is formed when a very large, cool cloud of dust collapses. Gravitational potential energy is released when the cloud collapses. It can be shown that the gravitational potential energy of a spherical cloud of mass *M* and radius *r* is given by $E=\frac{3GM}{5r}$.

Estimate the radius of a planet of density $\rho=6\times10^{3}\text{ kg m}^{-3}$ if the energy released in making the planet is enough to increase the temperature to 2000 K (so that rocks and metals melt). Take $c=1\times10^{3}\text{ J kg}^{-1}\text{ K}^{-1}$.

The original cloud has energy $E=\frac{3GM^2}{5R}$ where *R* is its enormous radius. The planet has energy $E=\frac{3GM^2}{5r}$ where *r* is the planet radius and so the difference of the two is the energy released in making the planet from the cloud. But *R* >> *r* and so, approximately, the energy released is just $E=\frac{3GM^2}{5r}$.

Then $\frac{3GM^2}{5r}=Mc\Delta\theta$ and so $r=\frac{3GM}{5c\Delta\theta}=\frac{3G}{5c\Delta\theta}\frac{4\pi r^3\rho}{3}$ so that finally:

$$r=\sqrt{\frac{5c\Delta\theta}{4\pi\rho}}$$

We have been told that $\Delta\theta=2000\text{ K}$. Assuming an earth like planet, $\rho=6\times10^{3}\text{ kg m}^{-3}$ and $c=1\times10^{3}\text{ J kg}^{-1}\text{ K}^{-1}$ we get an estimate of

$$r=\sqrt{\frac{5\times10^{3}\times2000}{4\pi\times6.67\times10^{-11}\times6\times10^{3}}}\approx10^{6}\text{ m}$$

which is not an unreasonable estimate.